

How do we measure the properties of stars?

(freely adapted from Elementary Astronomy lecture course given at Pennsylvania State University, original notes written by Professor Robin Ciardullo).

Parallax is rather a useful effect when studying stars, it can tell us how close the nearest stars are. The motion of the Earth around the Sun provides the parallax and we can measure the parallax angle. For the nearest stars, this angle is about one arcsecond, that is, $\frac{1}{60}$ of $\frac{1}{60}$ of one degree ($\frac{1}{3600}$ of a degree). For comparison, the angular size of the Moon is $\frac{1}{2}$ of a degree. Thus, if you divide the Moon into 1800 pieces, you'll get 1 arcsecond.

Through the trigonometry of the extremely skinny triangle on the right, we can then measure a distance.

Astronomers always try to make things as easy as possible for themselves. They define a new unit of distance, called the **parsec**.

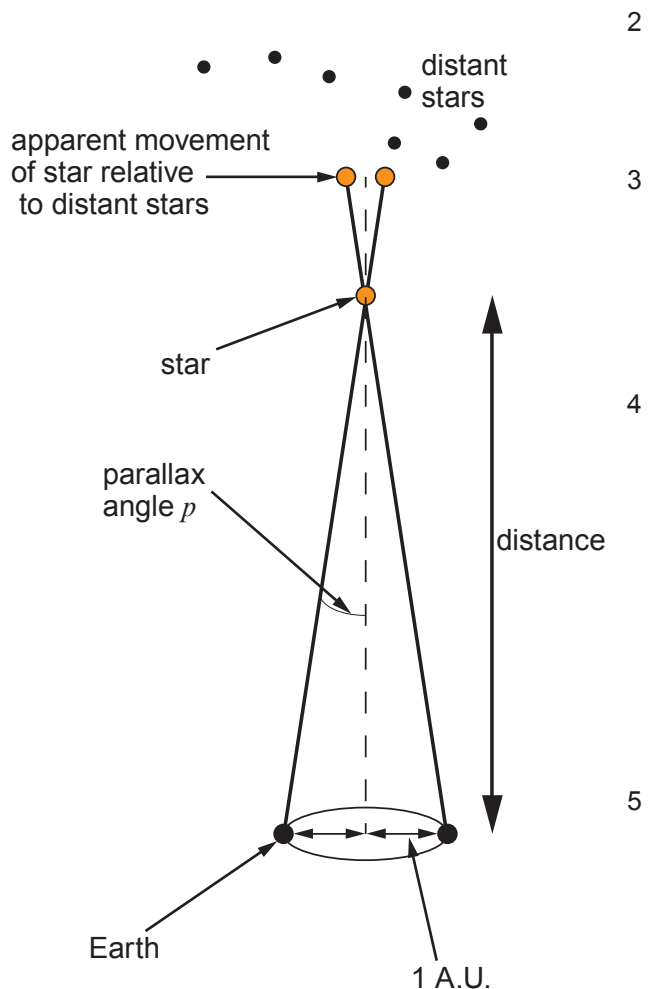
One parsec is **the distance a star would have if it had a parallax angle of 1 arcsecond**. (For those who insist on numbers, it's about 206 265 Astronomical Units, or 3×10^{13} km or 3.25 light years.) Note that by defining a parsec in this way, the relationship between the measured parallax angle and the implied distance is trivial. If a star has a parallax of 1 arcsec, its distance is 1 parsec. If it has a parallax of $\frac{1}{2}$ arcsec its distance is 2 parsec. If it has a parallax of $\frac{1}{10}$ arcsec, its distance is 10 parsecs. And so on.

If we measure the distance to a star, and we measure how bright a star appears in the sky, we can then figure out the star's intrinsic brightness i.e. how bright the star really is. (Is the star a matchstick in front of our eyes, or a searchlight far away?) This is done using the inverse square law of light, which relates the intrinsic luminosity, L to its apparent luminosity I by:

$$I = \frac{L}{r^2} \quad \text{where } r \text{ is the distance.}$$

Astronomers don't measure brightness in terms of watts (or even gigawatts). Sometimes they use **solar luminosities**, which is how bright a star is compared with the Sun. (For reference, the Sun is equivalent to a 4×10^{26} W light bulb!) More often (unfortunately), they use **absolute magnitude**.

Apparent magnitude describes how bright a star appears in the sky. Vega's apparent magnitude is defined as zero. Deneb's apparent magnitude is close to 1; Polaris (the North Star) is close to 2. The faintest star you can generally see has an apparent magnitude of around 3 or 4. The faintest star you can see out in the desert far from the lights of a city has an apparent magnitude of 6. Note that the magnitude scale goes backward – big numbers represent faint stars. Also note that magnitude scale works the way the human eye does,



which is logarithmically. This means that each magnitude is 2.5 times fainter than the previous magnitude. A star with apparent magnitude $m = 1$ is 2.5 times fainter than a star with $m = 0$. A star with $m = 2$ is $2.5 \times 2.5 = 6.25$ times fainter than one with $m = 0$. And so on. In fact, a difference of 5 magnitudes is defined as being equivalent to a factor of 100 in brightness.

The apparent magnitude of a star describes how bright the star appears. The absolute magnitude of a star describes the star's intrinsic brightness. Absolute magnitude is defined in this way – **a star's absolute magnitude is the apparent magnitude the star WOULD have IF it were at a distance of 10 parsec.** The Sun has an apparent magnitude of -26 . However, if the Sun were 10 parsecs away from the Earth, it would be much fainter; its apparent magnitude would be about 5 (you wouldn't normally be able to see it). So the Sun's absolute magnitude is 5. For stars other than the Sun, the formula that relates apparent magnitude, absolute magnitude and parallax angle is:

$$M = m + 5(1 + \log_{10} p)$$

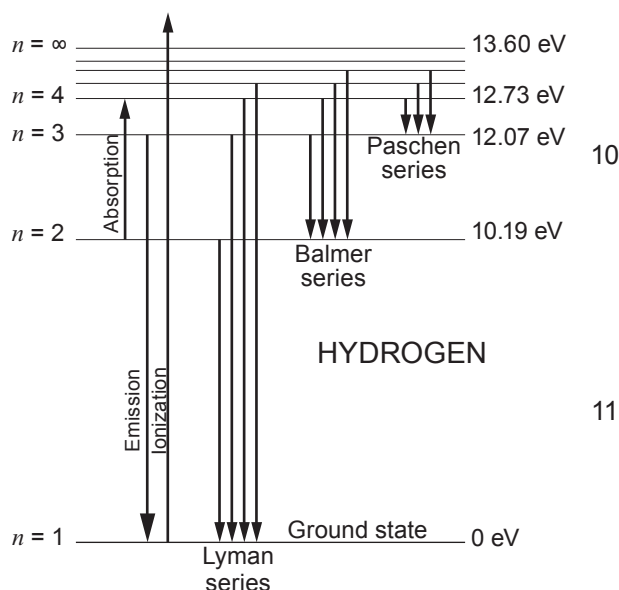
where M is the absolute magnitude, m is the apparent magnitude and p is the star's parallax angle in arcseconds.

Stellar surface temperatures are relatively easy to estimate. One way to do this is to estimate the star's colour. The redder the star, the cooler it is; the bluer the star, the hotter it is. But there's another way to measure a star's temperature, which works even when (for one reason or another), the colour method doesn't work well. The method involves looking at the star's spectrum.

Stars are made mostly of hydrogen and helium. Nine out of ten atoms in the universe are hydrogen atoms. Nine out of ten of what's left is helium. So, when we observe the spectrum of a star, we should see mostly hydrogen and helium. We don't, and the reason comes from the atomic physics of the individual elements.

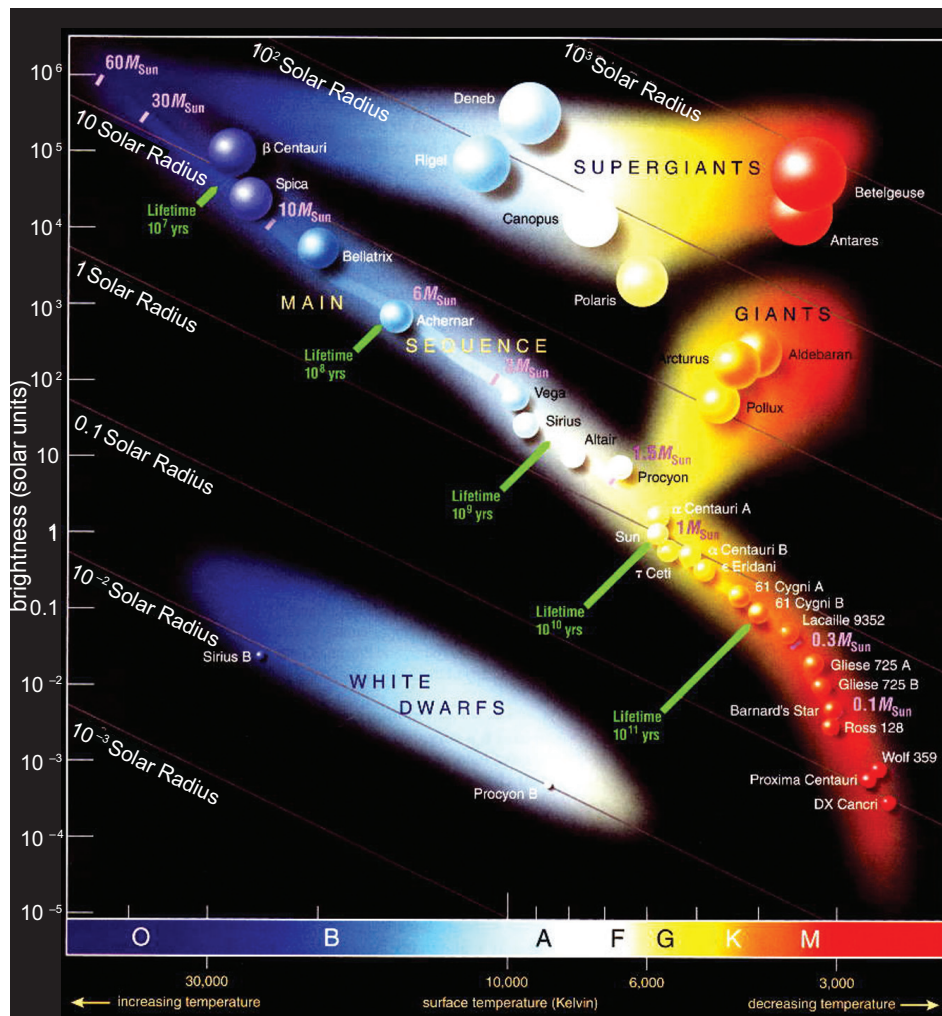
To understand this, let's consider the hydrogen atom as an example. Like all atoms, hydrogen has multiple levels for its electron. It turns out that for hydrogen, the distance between the first level and the second level is huge – it's equivalent to a far ultraviolet photon. Optical absorptions for hydrogen only occur when an electron in the second level grabs a photon and goes up to a higher level. This atomic structure has an interesting consequence.

Consider hydrogen in the atmosphere of a cool, red star. Virtually all the hydrogen will have its electrons in the ground (lowest) state. In order for one of these electrons to be in the second level, where it can grab an optical photon, it either has to (a) absorb an ultraviolet photon of the proper energy, or (b) be hit by something that has enough energy to push it up. But, in the case of a cool star, there are hardly any ultraviolet photons to absorb, and the atoms are moving so slowly that none of the collisions are hard enough to move an electron to a higher energy level. As a result, in cool stars, there are no hydrogen atoms that have their electrons in the second level, and therefore there are no **optical** absorption lines from hydrogen.



Now consider a very hot star. This hot star emits many high energy photons, and many of these are energetic enough to kick a hydrogen electron completely out of the atom (i.e. to ionize the atom). If all the hydrogen atoms have lost their electrons, then there won't be any hydrogen electrons in the second energy level, and again, there will be no optical absorption from hydrogen. So hot stars, like cool stars, will show no hydrogen absorption lines in the visible range. Thus, if one sees strong hydrogen absorption, the star must be of intermediate temperature (it is strongest at about 10 000 degrees).

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So far, we have discussed how to derive the intrinsic luminosity of a star and the star's temperature.

When astronomers find out two properties of an object (such as a star), the first thing they usually do is plot one versus the other. This is the **Hertzsprung-Russell diagram**, otherwise known as the HR diagram. The x -axis of the HR diagram gives the stars' temperatures (or, equivalently their colour, or spectral type).

Because astronomers like to do things backwards, hot blue, O-stars are plotted on the left, and cool, red, M-stars are plotted on the right. The y -axis of the HR diagram gives the stars intrinsic luminosity (or absolute magnitude). Bright stars are at the top of the diagram, cool stars are near the bottom. An

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overwhelming majority of stars (more than 90%) fall in a band across the diagram, going from cool and faint to hot and bright. This band is called the **main sequence** and it makes sense. Recall that from the blackbody law, hot objects radiate a lot more energy than cool objects (by temperature to the fourth power). So, it is reasonable that cool stars are faint and hot stars are bright. However, there are some peculiar stars that do not fall on the main sequence. In particular, some stars are both very red and very bright, while others are very blue and very faint. Let's consider the red stars first. Each square metre of these stars must be relatively faint, since cool objects do not radiate much light. The only way that these stars can be bright is to be enormous (i.e. they must have a lot of square metres). We will call these **red giant stars**. Conversely, the only way very hot stars can be faint is to be exceedingly small. These are **white dwarf** stars (though a better name would be blue dwarf stars).

Note that in reality, the brightness of a star depends on two factors: its temperature and how much surface area the star has. In other words:

$$L = br^2 T^4 \quad 15$$

where L is the absolute luminosity of the star, r is the star's radius, T is the star's surface temperature, and b is a number to make the units come out all right. Note also that the radius of the star is plotted with a dashed line in the HR diagram – the lower left (blue) of the diagram is small, while the upper right (red) end of the diagram is large.

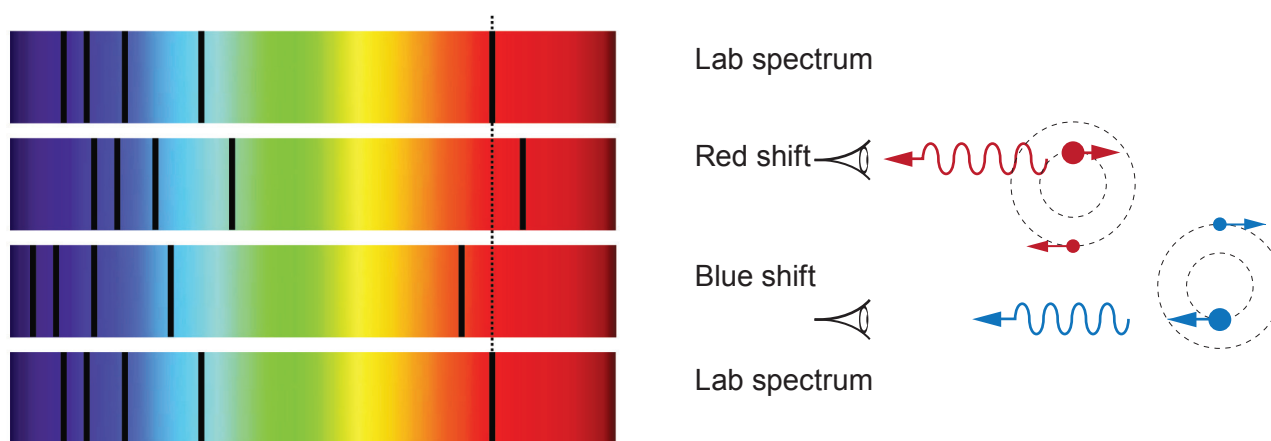
Another important property to understand is stellar mass. Stars can be 'weighed' using the laws of Kepler and Newton. Recall that Newton's modification to Kepler's 3rd law is:

$$(M + m)T^2 = a^3 \quad 16$$

where T is the orbital period (in years), a the semi-major axis of the orbit (in astronomical units), and M and m the masses of the two objects (in solar masses). If we could identify **binary stars** that orbit around each other, and if we could measure their period and their semi-major axis, we could measure their total mass.

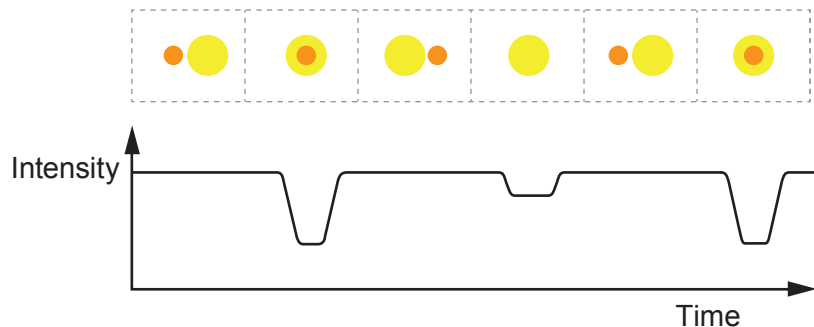
The universe is kind to us. Perhaps $\frac{1}{2}$ of all stars are binary stars. Astronomers have different names for these stars, which depend on how we identify and perceive their binary nature. Here's a list of terminology; note that the same stars can occasionally fall into more than one category.

Spectroscopic Binary. This is an extremely important type of binary star. For spectroscopic binaries, the two stars are so close together that an astronomer only sees one object (usually only the brighter of the two stars). However, over time, the astronomer will see that the Doppler shift changes. First, the star will be moving towards us; then away from us; then towards us again. Spectroscopic binaries can have periods of months, days, hours, or even minutes!



Spectroscopic binaries are important because of what they can tell us. First, by monitoring the Doppler shift, one can time how long it takes to complete one forward-backward-forward cycle. That's the star's period. Next, again, from the Doppler shift, one can determine the star's velocity. Velocity times time equals distance, so this gives you the size of the star's orbit, and, with a minimal amount of mathematics, the orbit's semi-major axis. The law of gravity then gives you the total mass of the two stars. 19

If one star is very bright, while the other is very faint, then you will only see the absorption lines of the bright star. In this case, the star is a **single-line spectroscopic binary** and you can't do any more with it. However, if both stars are about equal brightness, then you may see absorption lines from both stars. While one star is moving towards you, the other will be moving away from you. The relative speeds of the stars tell you the relative mass: the more massive star will be moving slowly, while the less massive star will be moving rapidly. In this **double-line spectroscopic binary** case, you can not only measure the total mass of both stars together, but the mass ratio of the stars. You thus have individual masses.

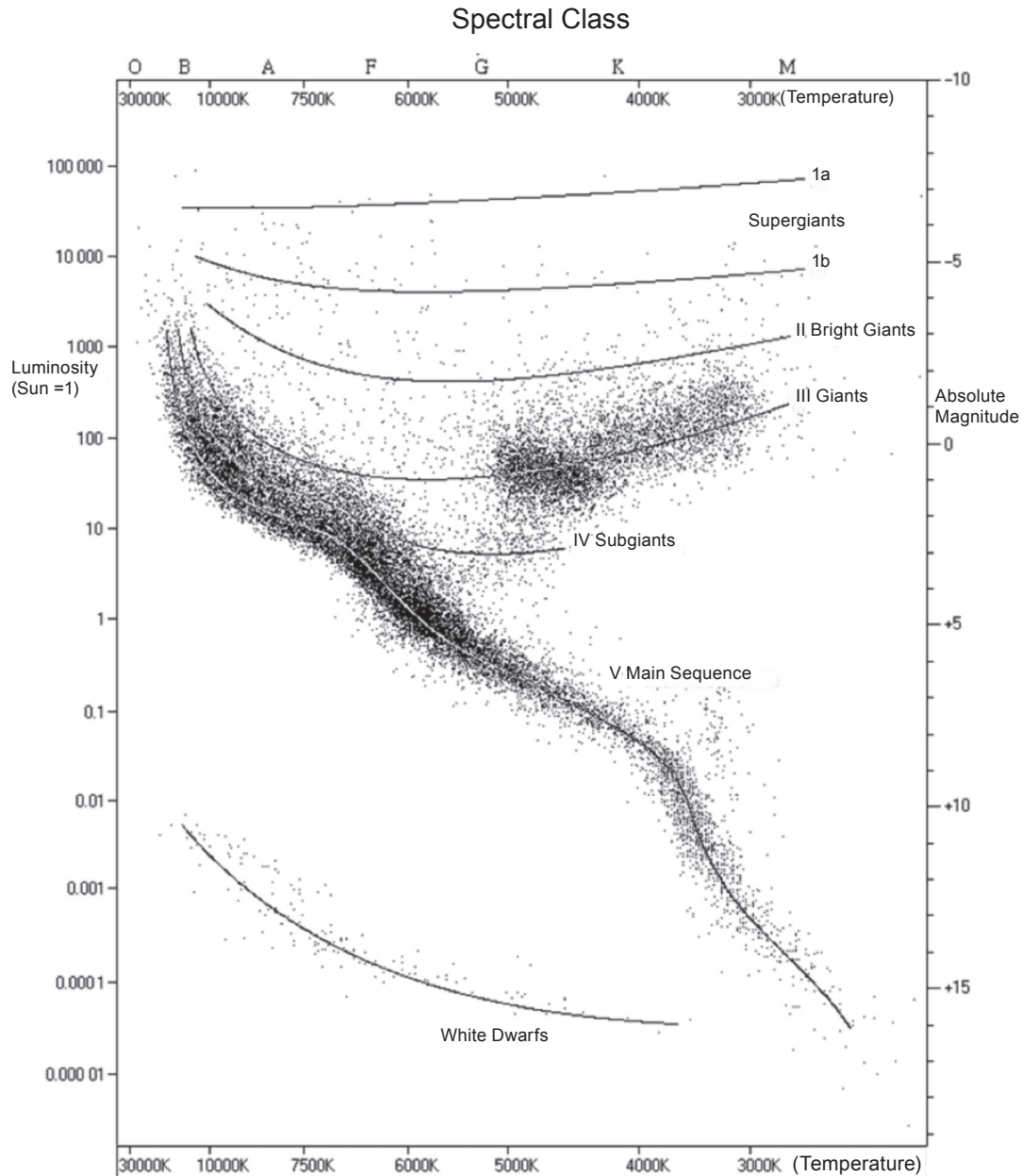


Eclipsing Binary. Eclipsing binaries are like spectroscopic binaries in that the astronomer only sees one object. However, in this case, one star occasionally gets in the way of the other, i.e. one star eclipses the other. When this happens, the light from the system decreases. By following the object's **light curve**, one can measure the star's period. Eclipsing binaries are important because for these stars, you know the orientation; the plane of the orbit is along your line-of-sight.

If you measure the absorption lines of an eclipsing binary, you'll find it to be a spectroscopic binary. Since the inclination of the stars is known, then there is no ambiguity about the mass determination from the Doppler shift. Moreover, by timing how long it takes for the stars to move into and out of eclipse, and by noting how fast the stars are moving, it is possible to use these systems to measure not only a star's mass but also its size.

Incredible though it may seem, each dot on the Hertzsprung-Russell diagram represents a nearby star. All this information on thousands of stars has been gathered using little more than the theories discussed in this article. The equipment used were basic mirrors, prisms (or gratings) and light detectors. However, let it not be forgotten the amount of work required to collate this quantity of data and the intelligence of the few with the insight to develop these theories in the first place.

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